Accurate Calibration of Vibration Transducers from 0.25 Hz to 11,500 Hz

Edward L. Peterson, P.E.
Director, Applications Engineering
MB Dynamics

Mark McElroy
Software & PC-based Systems Development
MB Dynamics

Compliance to international quality standards (ISO 9000) has heightened awareness of calibration and traceability. This emphasis has focused more attention on the fields of metrology and calibration, creating both new concerns and opportunities for those involved in all aspects of test and measurement activities including vibration transducers such as accelerometers and velocity pickups. In addition to a general awareness of the need to calibrate accelerometers and velocity pickups, there is a growing special interest in very low frequency and high frequency calibrations. MB Dynamics has developed a system that allows accurate calibration of vibration transducers from 0.25 Hz to 11,000 Hz using a reference accelerometer, a more affordable alternative to lasers.

Introduction

Historically, calibrations of vibration transducers have been between a low frequency of 5 Hz to 10 Hz and a high frequency of 4000 Hz to 5000 Hz. In recent years, increasing demand has been made to calibrate below 1 Hz, in some cases as low as 0.25 Hz, and at higher frequencies to 8,000 Hz, 10,000 Hz or even 11,000 Hz.

In concert with this need to calibrate to lower and higher frequencies is the most important factor of good calibrations – *assuring reasonable accuracy.* The challenge has been for developers to provide solutions that are economically feasible. Some labs have taken the approach of using lasers, which have some strong technical attributes, but are extremely expensive. Using a precision calibrated reference accelerometer (REF) is a much more affordable approach. Historically however, the REF approach has run into limitations at low frequencies and high frequencies. The challenge for this project, therefore, was to push the limits of traditional REF technology to see how accurate it could be at both low and high frequencies.

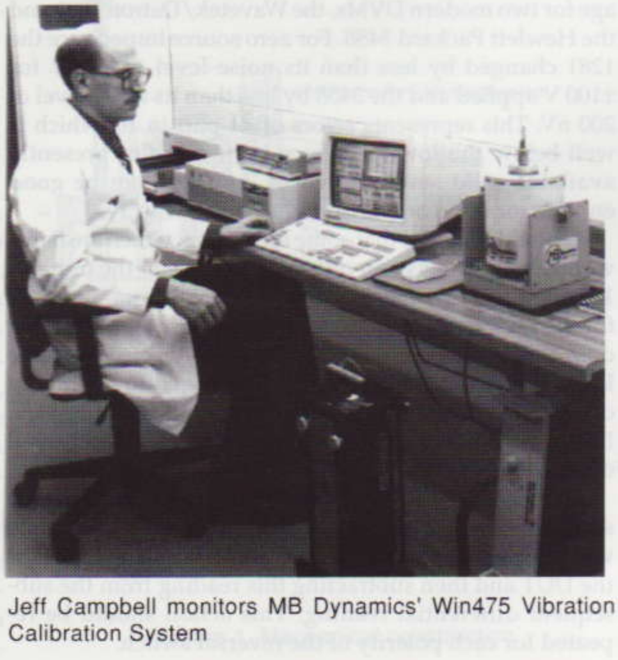
Signal Level Challenges of Ultra-Low Frequency Accelerometer Calibration

A major problem at ultra low frequencies is that acceleration values become extremely small as the following equation demonstrates:

$$g = 0.0511\, f^2 d$$

where, g= acceleration (gpeak), f= frequency (Hz), d= displacement (inches pk-pk).

Acceleration gets smaller at a frequency squared rate if you hold a constant displacement and go to lower and lower frequencies.



Jeff Campbell monitors MB Dynamics' Win475 Vibration Calibration System

This effect is shown graphically in Figure 1. For example:

- At 10 Hz, 0.5 inches pk-pk of displacement produces 2.56 g's pk of acceleration;
- At 1 Hz, 0.5 inches pk-pk of displacement produces only 0.0256 g's pk of acceleration; and
- At 0.25 Hz, 0.5 inches pk-pk of displacement produces only 0.00160 g's pk of acceleration.

It will be shown later that 7 mV (or 7 pC) is a good target level for the nonamplified output of a transducer. This level of output from the transducer is necessary to achieve sufficient accuracy. The graph in figure 1 shows that a 10 mV/g (or 10 pC/g) accelerometer produces over 25 mVpk of output above 10 Hz, which is significantly higher than the targeted 7 mVpk of output, and therefore ensures accurate measurements.

However at 1 Hz, 0.5 inches pk-pk of displacement produces only 0.25 mVpk when a 10 mV/g accelerometer is used. This is far short of the 7 mVpk target. In fact, even a 100 mV/g accelerometer does not produce enough output at 1 Hz with a displacement of 0.5 inches pk-pk. To accurately calibrate a 100 mV/g device at 1 Hz requires approximately 1.4 inches pk-pk of displacement to produce the target 7 mV/g (or 7 pC) pk of output.

Since 0.25 Hz is four times lower in frequency than 1 Hz, the low acceleration level problems are sixteen times worse (the *square* of the frequency ratio). A 100 mV/g accelerometer would need to be vibrated with an amplitude of approximately 22 inches pk-pk to produce the target 7 mV/g of output. Clearly this is not practical. However accelerometers designed to measure vibration at low frequencies always have high outputs for the very reasons established in the preceding discussion. A 500 mV/g accelerometer can produce sufficient output when vibrated with an amplitude of 4.4 inches pk-pk of displacement (which *is* practical).

The other challenge faced by the practitioner doing low frequency accelerometer calibrations is the frequency response of the standard or reference accelerometer. Historically, low impedance or high impedance piezoelectric type accelerometers have been used for accelerometer calibration systems. These devices can be made to respond at low frequencies if care is taken to assure the time constant of the signal conditioning is matched to the required frequency response of the measurement and measurement sensitivity is high (≥ 500 mV/g). Additionally, the accelerometer linearity, measurement noise, and hysteresis must be low enough to assure that the signal acquired by the device provides adequate headroom against these combined noise floor items.

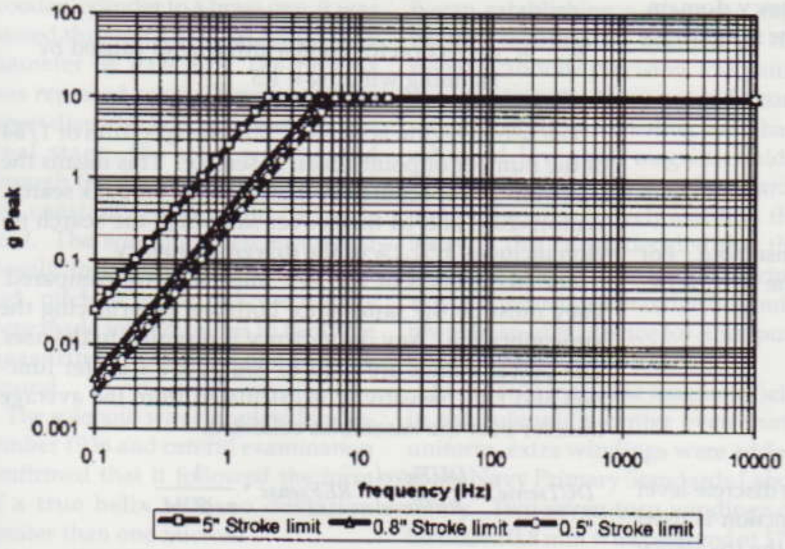


Figure 1. Displacement and Acceleration

Other types of accelerometers that exhibit good low frequency response are capacitive, piezoresistive, and servo accelerometers. These devices can produce high outputs down to DC and sensitivities of 500 mV/g or greater.

Getting the Best Results from Difficult Measurements

The problems that occur with ultra low frequency signal levels can be mediated by using a device that provides large displacements, high output sensitivities, and good frequency response. The design elements used by the MB Dynamics development team to ensure these characteristics included:

1. A low noise signal conditioner to minimize noise consisting of the following:

- Low noise amplifiers;
- Hi pass filters set below 0.1 Hz; low pass filters above 11.5 kHz;
- Massive ground planes to assure common signal reference through circuit;
- Low noise power supply;
- Signal conditioning of both sensors by one packaged unit using one power supply and ground plane;
- Noise floor below 10 µV.

2. Gains set to optimize system resolution. The data acquisition is performed using a 12 bit A/D converter. Since maximum accuracy is desired, it is necessary to adjust the gain. The signal conditioner and the A/D board gains should be set automatically by software. The MB Dynamics system sets the following gains: 0.5, 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, and 2000. This allows input voltage peak-to-peak ranges from 2.5 mV to 10 V. The gain setting is independent for each channel, allowing high sensitivity references simultaneously with low sensitivity test accelerometers. Gain levels are set prior to drive signal output based on estimated peak voltages. When the selected vibration level is reached, the gain for each channel is reevaluated and adjusted accordingly. This is determined by the maximum gain setting that does not produce clipping.

3. Oversampling to ensure good spectral line spacing in the frequency domain and a high confidence level. At each test level and frequency point, data collection is started after vibration frequency and levels are confirmed and gain is set. The data is collected in ensembles of 2048 points at 64 times the driven frequency. Each ensemble contains 32 cycles of data.

The collection of data at many times the driven frequency is called over-sampling. Oversampling allows the system to be more tolerant of harmonic distortion at low frequencies and using more data points gives the system better spectral resolution. In this system it is possible to set the number of ensembles used per measurement to 2, 4 or 8. The more ensembles taken, the more averages and higher degrees of freedom. However, taking more ensembles requires more time. The 4-ensemble setting is recommended since it gives sufficient accuracy while minimizing acquisition time. In the frequency domain, the spectral line spacing is determined by the sample rate and the number of points in the ensemble.

Frequency Interval (fi)

$$fi = \frac{SampleRate}{2n}$$

where, n is the number of points in an ensemble. For example, if the sample rate is 64 times the driven frequency (f), then:

fi = 32f/n.
If n = 2048,
then fi = 1.6% of f.

4. Data is "windowed" for more accurate discrete level measurements. The Hanning window function is used because of its excellent peak resolution in the frequency domain.

Hanning Window :

$$X(i) = R(i) * 0.5 - R(i) * 0.5 * \cos\left(\frac{2\pi * i}{n}\right)$$

5. Processing is performed in the frequency domain. The frequency domain process is more accurate and noise tolerant. Additionally, it has the added benefit of significantly reducing any errors due to sine wave distortion caused by the shaker. The process begins with the individual spectra for the REF and the DUT. The Auto Power Spectrum A(ω) is calculated for each channel.

Auto Power Spectrum:

$$S_{xx} = FFT(X)FFT^{*}(X)$$

Scaled Magnitude Spectrum:

$$A(\omega) = \frac{|S_{xx}(\omega)|}{n^2}$$

The frequency and peak power are measured for each channel. This is accomplished by a peak search and interpolation routine.

$$frequency = \frac{\sum_{j=i-n/64}^{i+n/64} [A(j)(j * fi)]}{\sum_{j=i-n/64}^{i+n/64} A(j)}$$

$$power = \sum_{j=i-n/64}^{i+n/64} \frac{A(j)}{b}$$

b = correction for bandwidth spread caused by Hanning window = 1.5

The search region is set to 1/32 of the spectrum or 1/64 of the number of points in an ensemble. This means the spectrum ± 32 spectral lines is included in the peak search. With n=2048 and 64 times over-sampling, the search region includes ± 51.2% of the driven frequency.

The frequencies of the REF and DUT are compared. They must be the same since both are experiencing the same vibration. Any discrepancy is reported to the user. The power values are used to compute a transfer function and DUT sensitivity is calculated from the average transfer over the ensembles collected.

$$DUTsense = \frac{DUTlevel}{REFlevel} * REFsense * \frac{1}{selfCAL}$$

where,

SelfCAL = the self calibration transfer function of the signal conditioner or the linear interpolated transfer;

REFsense = the NIST traceable sensitivity or the linear interpolated sensitivity;

DUTlevel/REFlevel = the average transfer function.

Detection and Correction of Errors in the Test System

Whenever possible the test system should detect errors it is causing and if possible, correct them. To insure accuracy in a single step test, the system makes corrections for the mismatching of the REF and DUT channels in the signal conditioner (figure 2). This is done through self-calibration.

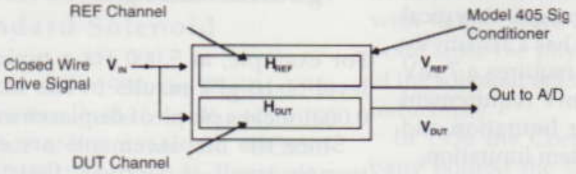


Figure 2. Closed-Wire Channel-to-Channel System Self Calibration

The system sets up an accelerometer calibration of a closed wire on the REF and DUT channels. The REF and DUT channels both measure the same signal and the transfer function should be 1.00 at all frequencies.

$$V_{REF} = V_{IN} H_{REF} \quad \text{and} \quad V_{DUT} = V_{IN} H_{DUT}$$

The REF to DUT transfer function is:

$$selfCAL = \frac{H_{DUT}}{H_{REF}}$$

If we assume that: $H_{REF} = 1$

then, $V_{REF} = V_{IN}$

and, $selfCAL = H_{DUT} = \frac{V_{DUT}}{V_{REF}}$

The actual transfer function will differ where the two channels are not exactly matched. This difference is typically less than 0.25 percent over the 11.5 kHz bandwidth. However, it is a consistent property of the measurement system and therefore is measured and applied as a correction factor in the calibration equation.

Coherence is measured to determine the quality of the signals and is used to mark suspect data. The coherence of the calibration should be ideally 1.000 at all frequencies used in the calibration. If the coherence is less than 0.950, then that frequency should be flagged as questionable. Coherence (K_{xy}) is defined as follows:

$$K^2_{xy}(\omega) = \frac{\left|S^2_{xy}(\omega)\right|}{S_{xx}(\omega) S_{yy}(\omega)}$$

When signal levels out of the sensors are 7 mVPk or larger, the overall system is able to produce excellent results. Therefore it was decided to flag any frequencies in a calibration that had less than 7 mVpk (7 pC for high impedance transducers.)

Reference or Standard Accelerometers

It is important to keep in mind that no accelerometer is perfect. That is, any accelerometer will have different sensitivities at each frequency in the spectrum. Therefore it is crucial that the calibration system be given the information on the frequency response characteristics of the reference accelerometer and that the test system correct for those known deviations in sensitivity at various frequencies. Using this approach makes it acceptable to use a reference accelerometer that rolls off at low or high frequencies - as long as it is stable (repeatable) and as long as it produces at least 7 mVpk of output.

The best way to obtain sensitivity data on the reference accelerometer is to have it calibrated at a primary standards laboratory. MB Dynamics uses the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland. This assures the best possible accuracy and traceability for calibrating accelerometers and velocity pickups.

Keep in mind that the turnaround time for sending a reference accelerometer off premises can be problematic. It is therefore good practice to keep two reference accelerometers on hand, one to use in the system and the other to be sent back to NIST for periodic recalibration.

The data is collected and processed at discrete frequencies and levels specified by the user in a sine test profile. The sine sweep can be stepped to speed the test process, *i.e.* the test can ramp up, collect data, and ramp down at each specified frequency and level in the profile.

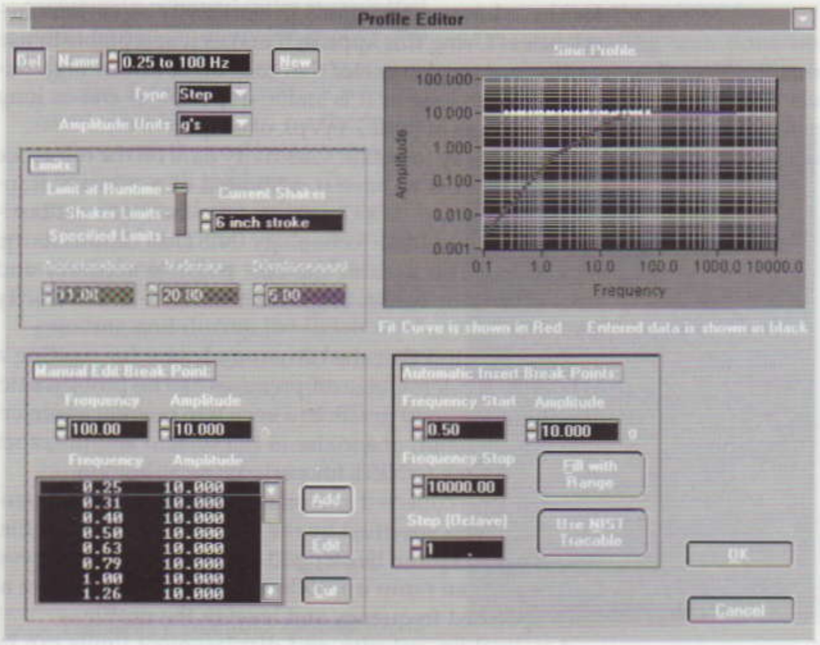
Acceleration, velocity, and displacement limits can be imposed on the test to protect shakers or sensors used in testing (figure 3). The profile can be specified in g's or ips, and the specified points are plotted on the graph and listed in the table. The profile can be quickly filled in octave or fractional octave steps or the REF traceable frequency points can be used. The frequencies at which the REF has been calibrated by NIST are the most important test frequencies because they are directly traceable to NIST. This is important because the resulting calibration of the DUT is directly traceable to the REF calibration certificate.

The 7 mV Rule

The instrumentation used for these calibrations include extremely high quality amplifiers, 12 bit ADCs, and low noise instrumentation throughout. However, it was determined through the course of the tests that when the nominal output of either accelerometer falls below 7 mVpk or approximately 5 mVRMS, the results of the Redstone tests (*i.e.*, the Overall system accuracy [OSA]) became unacceptably large.

The basis for this rule is a signal-to-noise issue. While the noise floor of the measurement system has a minimum level of 10 V, the noise floor of the sensor is much higher. All sensors have either a resolution specification or a signal-to-noise specification. The result is that the sensor always outputs some noise level. Any mechanical acceleration signal that is small enough to approach that noise level will not be resolved by the sensor. Therefore the signal level must always be above the noise.

As a rule, it is minimally acceptable to have an order of magnitude that is more signal than noise (*i.e.*, 20 dB of minimum dynamic range). A typical vibration transducer has a sensitivity and noise floor that requires a 7 mV peak signal. The 7 mV requirement is therefore a sensor limitation and not a calibration system limitation.



Name: 0.25 to 100 Hz
Type: Step
Amplitude Units: g's
Current Shaker: 6 inch stroke
11.00 20.00 5.00

Manual Edit Break Point
Frequency: 100.00 Amplitude: 10.000

Frequency	Amplitude
0.25	10.000
0.31	10.000
0.40	10.000
0.50	10.000
0.63	10.000
0.79	10.000
1.00	10.000
1.26	10.000

Automatic Insert Break Points
Frequency Start: 0.50 Amplitude: 10.000 g
Frequency Stop: 10000.00
Step (Octave): 1

Figure 3. MB Dynamics Win475 screen of sine test profile.

Special Problems Encountered at High Frequencies

Above 3000 Hz the practitioner is confronted with a different set of challenges. Two major limiting factors are attachment of the accelerometers to the shaker and mass loading. At high frequencies the displacements can become extremely small. Rearranging the equation presented at the beginning of this article:

$$d = g/0.0511\, f^2$$

d = displacement (inches pk-pk)

where,

f = frequency (Hz)
g = acceleration (gPk)

For example, at 5,000 Hz a typical level of 10 gPk results in less than 0.0001 inches pk-pk of displacement.

Since the displacements are extremely small, it is crucial that the mating surfaces be clean and smooth. It is also necessary to lightly lubricate the mating surfaces and use a torque wrench to attach the REF, device under test (DUT), and any fixturing required to mount either to the shaker or to the other device. Note that the above procedures apply to *both* the REF and the DUT. Failure to assure good mating can result in mechanical chatter due to looseness induced by the poor mounting technique or induced base strain which can appear as data on the acceleration waveform.

There are basically two kinds of REFs, as shown in figure 4. A "double ended" REF is shown on the right in this figure. This kind of REF is attached to the shaker table and the DUT is then attached to the top of the REF. In this configuration the published sensitivities of the REF are only valid for one particular DUT weight (*e.g.*, 50 grams). DUTs weighing less (or more) than the prescribed weight will cause the REF to have different sensitivities. Below 2000 Hz this er-

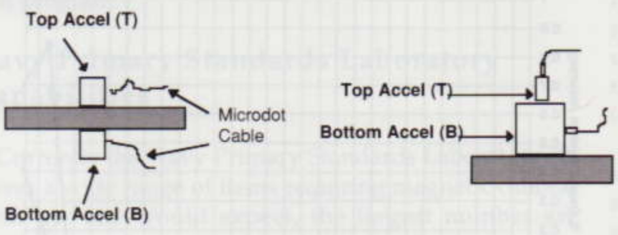


Figure 4. Reference (REF) accelerometer setups.

ror is minimal (typically less than 0.25%). However at frequencies in the 5,000 - 11,000 Hz range, these errors in REF sensitivities can get extremely large. For example, if a double-ended REF PCB Model 301A04 is used with out a DUT (*i.e.*, the REF is used inside a shaker's armature and the DUT attached to the top of the armature) the error can be more than 10% at 10,000 Hz. Obviously, this particular configuration should be avoided!

The Redstone Check

In order to evaluate the effectiveness of the signal processing techniques, data manipulation routines, and any deficiencies of the shaker subsystem, it was necessary to perform some type of verification. One of MB Dynamic's first generation calibration systems had been certified by the metrology lab at Redstone Arsenal. MB's development team was impressed with the simplicity and effectiveness of Redstone's verification method and has since adopted it.

The Redstone Check uses two accelerometers, B (bottom) and T (top). It works equally well for either of the two basic configurations (figure 4). A NIST traceable reference accelerometer (accelerometer B) is used as a comparison standard to calibrate accelerometer T. The sensitivity data is collected on accelerometer T to redefine it as a reference accelerometer and use this data to calibrate accelerometer B.

Theoretically, the sensitivities obtained for accelerometer B should be exactly what was started with. Any differences in sensitivities between what was started with for accelerometer B and what was returned at the end of the second step are due to errors in the calibration system itself.

The derivation shown in figure 5 defines the relationship of the total measured error to the overall system accuracy (OSA) for one direction. The setup in figure 5 assumes that a noncritical voltage of a magnitude V_{in} is applied to both the reference and DUT channels simultaneously. Each of the channels has its own characteristic error term, R for the reference channel and D for the DUT channel.

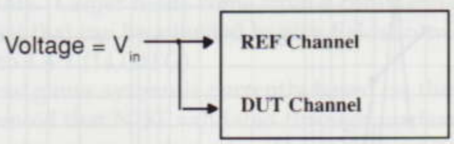


Figure 5. Overall system accuracy derivation

The voltage reported by the system for that measured in the reference channel is therefore:

$$V_{REF} = V_{in}[1+\Sigma_R]$$

Similarly, the voltage reported by the system for the DUT channel is:

$$V_{DUT} = V_{in}[1+\Sigma_D]$$

The DUT sensitivity is calculated as a ratio of these terms, as modified by the known reference sensitivity. If we assume a reference sensitivity of 100 mV/g, then the DUT sensitivity may be calculated as follows:

$$\text{DUT SENS} = V_{DUT}/[V_{REF}/\text{REFERENCE SENS}]$$

by substitution:

$$\text{DUT SENS} = V_{in}(1+\Sigma_D)/[V(1+\Sigma_R)/100\ \text{mV/g}]$$

or

$$\text{DUT SENS} = 100\ \text{mV/G}\ [(1+\Sigma_D)/(1+\Sigma_R)]$$

The value of DUT SENS in the Redstone verification process now becomes by substitution the value used for the reference sensitivity. The process of calculating a new DUT SENS is then conducted in identical fashion to that detailed above, *i.e.*,

$$\text{DUT SENS} = V_{DUT}/[V_{REF}/\text{REFERENCE SENS}]$$

Again, by substitution:

$$\text{DUT SENS} = V_{in}(1+\Sigma_D)/[V(1+\Sigma_R)/100\ \text{mV/g}(1+\Sigma_D)/(1+\Sigma_R)]$$

which reduces to:

$$\text{DUT SENS} = [(1+\Sigma_D)^2/(1+\Sigma_R)^2]100\text{mV/g}$$

Thus, the data provided by the Redstone Check results in the second test, which is the square of the actual error term in question. The net result is that a second test error

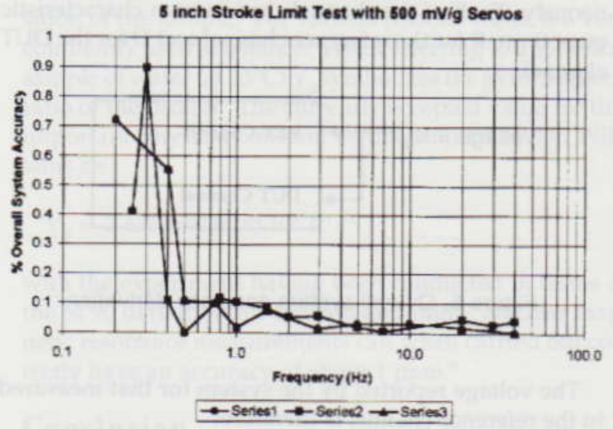


Figure 6. Scatter results for test range 0.25 Hz to 40 Hz with a 5 inch Stroke Limit

term of +0.5% reflects in fact an overall system accuracy error of $(1+0.005)^{1/2}$ or 0.2497%.

Typical Results

Actual calibrations were performed using a shaker capable of producing 5" pk-pk of displacement and using a servo accelerometer as the reference having a nominal 500 mV/g output. The device under test (DUT) was a servo accelerometer with a nominal sensitivity of 500 mV/g. The Redstone Check was performed several times to define the amount of scatter in the data, and the test was run from 0.25 Hz to 40 Hz. The results are shown in figure 6. It can be seen that in the range of 0.2 to 0.4 Hz the maximum error is 0.9%, whereas in the range 0.5 to 1.0 Hz the maximum error is less than 0.25%.

High frequency calibrations were performed using a 0.5 inch stroke limit, a 100 mV/g reference accelerometer, and a 100 mV/g DUT. This test was performed in the 10 Hz to 11,500 Hz range and the Redstone Check was performed several times to define the overall system accuracy. Figure 7 shows the results.

Many commercially available shakers have a 1 inch stroke limit. Typically, shakers in this category have flexures so it is advisable to use a bit less than full stroke when possible (this increases the life of the flexures and reduces the amount of sine wave distortion). It is also worth noting that some 1-inch stroke shakers can be driven up to 10,000 Hz. This configuration provides an uncommonly broad frequency range on one shaker. It was, therefore, decided to define the minimum frequency that could be expected if 0.8 inch pk-pk of displacement were available from the shaker. Figure 8 shows the results of 3 separate runs with this configuration. The data suggest that 2 Hz is a reasonable low-frequency limit for such a shaker.

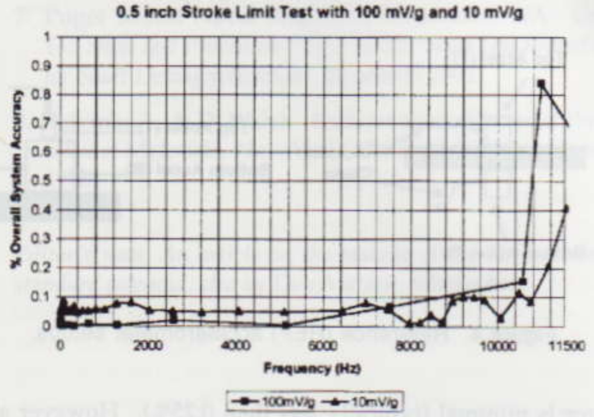


Figure 7. Results for test range 10 Hz to 11,500 Hz with a 0.5 inch Stroke Limit.

Conclusion

It is not necessary to use a laser to do accurate calibrations below 1 Hz. A traditional reference accelerometer can be used to obtain 1% OSA down to 0.25 Hz and 0.25% OSA down to 0.5 Hz. Figure 9 outlines the OSA limits for the 0.25 Hz to 11,500 Hz bandwidth.

Acknowledgments

The authors wish to acknowledge those individuals who provided sound technical expertise and equipment: Bill Rusen, P.E., Director of Product & Systems Engineering for MB Dynamics was instrumental in the design and assembly of test rigs used and is responsible for deriving the OSA formulation described in the Redstone Check; Larry Knight, Systems Integration for MB Dynamics, was

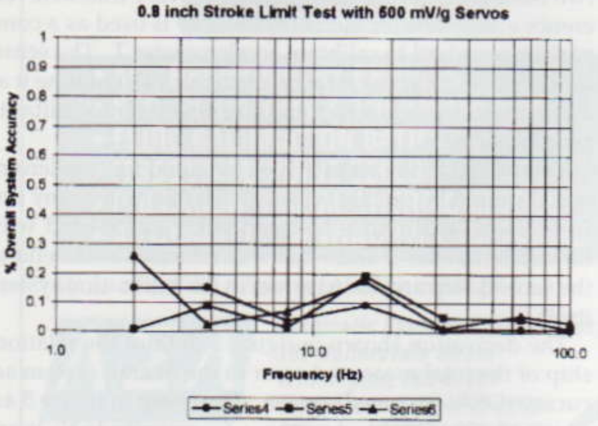


Figure 8. Results for test range 2 Hz to 100 Hz with a 0.8 inch Stroke Limit

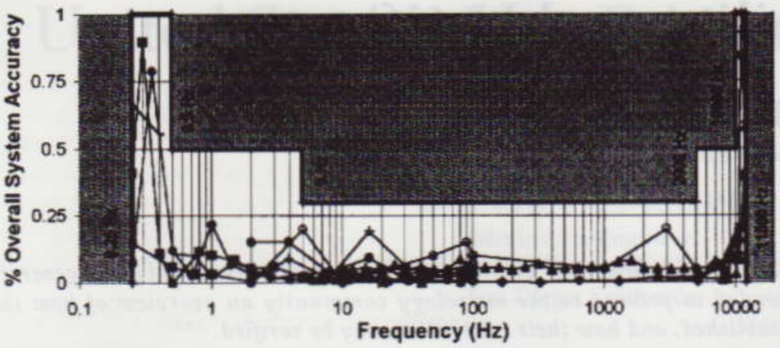


Figure 9. MB Dynamics Win475 overall system accuracy with typical data

very helpful in debugging physical tests throughout this project; and finally, Dave Jaquish of Boeing Airplane Company, who inspired this paper after receiving one of the first Win475 *Windows-based Vibration Transducer Calibration Systems from MB Dynamics. Boeing's desire for an ultra-low frequency calibration system inspired much of the research documented in this article.*

About the Authors

Edward L Peterson, P.E., has been part owner and Director of Applications Engineering for MB Dynamics in Cleveland, Ohio since 1985. Mr. Peterson handles applications covering the full range of experimental dynamics from modal analysis through design qualification tests and stress screening. During 1971-1985 Mr. Peterson held numerous technical and test management positions with Structural Dynamics Research Corporation, both in the States and the U.K. He holds both a BSME and MSME from the University of Cincinnati, and is a licensed Professional Engineer in the State of Ohio.

Mark McElroy is an MB Dynamics software engineer and is directly responsible for much of the development and testing refinement of MB's *Win475* Windows-based Vibration Transducer Calibration System. He will soon complete his BSEE at Case Western Reserve University.