Sources of Uncertainty in Standard Resistors

Part I: Alpha and Beta Coefficients and Their Effect on the Calibration Process

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As metrologists are acutely aware, when one transfers a measured value such as resistance, they also transfer a measurement uncertainty that is associated with the calibration of the standard resistor. Most metrologists have done an excellent job in understanding and accounting for the measurement uncertainty associated with the calibration of standard resistors when calibrating lower echelon standards. However, many users of standard resistors are not aware of the additional uncertainty that is added into the calibration process due to some of the fundamental physical characteristics of resistors.

Introduction

Standard resistors are commonly found in metrology labs today. These resistors are typically our most accurate resistance standards and are used to calibrate other laboratory standards such as high accuracy digital multimeters and multifunction calibrators. The standard resistors' measured value is used to characterize a resistance parameter on a unit under test, providing a critical link in the path of measurement traceability from working laboratory standards to higher echelons of measurement, such as the National Institute of Standards and Technology.

As metrologists are acutely aware, when one transfers a measured value such as resistance, they also transfer a measurement uncertainty that is associated with the calibration of the standard resistor. Most metrologists have done an excellent job in understanding and accounting for the measurement uncertainty associated with the calibration of standard resistors when calibrating lower echelon standards. However, many users of standard resistors are not aware of the additional uncertainty that is added into the calibration process due to some of the fundamental physical characteristics of resistors. Often the metrologist is aware of these physical characteristics, but fails to compute an estimate of measurement uncertainty due to these characteristics, believing that the computed uncertainty would not be significant to their process. This series will review some of the physical characteristics of standard resistors and will work through some examples of uncertainty associated with these characteristics.

How Accurate Should My Process Be?

This is the first question that all metrologists must consider when developing their calibration processes. Process uncertainty requirements may be driven by quality system requirements such as ANSI/NCSL Z540-1-1994. This quality document requires the maintenance of a 4:1 ratio between the accuracy of the standard to the accuracy of the unit under test when developing calibration procedures. Statistically derived quality levels for the calibration process may also drive uncertainty requirements for the calibration procedure. Regardless of the origination of the requirement, it is imperative that the measurement process uncertainty for one's standards does not exceed the minimum standards requirements listed in the unit under test calibration procedure.

The quality of the calibrations performed on test equipment is absolutely dependent upon keeping the measurement process uncertainty within the minimum standards requirements. Examples of minimum standards requirements for two instruments commonly associated with calibration labs are outlined below.

The first example of an instrument that requires a high quality resistor during its calibration is the HP 3458A Digital Multimeter. The HP 3458A accuracy specifications are relative to the calibration standards.[1] This is to say that the absolute accuracy of the HP 3458A is the relative accuracy specification as listed in the HP specification documents plus the accuracy of one's standard resistor and the uncertainty associated with the process of transferring the value of the standard resistor in the calibration procedure. Footnote 3 of the HP 3458A resistance accuracy specifications,[1] states to "Add 3 ppm of reading additional error for HP factory traceability of 10 kohm to NIST." (Note: If you are sending your HP 3458A out for calibration, do you know what the additional error for traceability of the 10-kohm standard is for your service provider?)

Another example of an instrument that requires precision resistance measurements is the Fluke 5700A meter calibrator. Many calibration laboratories perform the artifact calibration for the Fluke 5700A since it only requires a one-ohm resistor, a 10-kohm resistor, and a 10-volt reference standard. The manufacturer has specified uncertainty limits for these standards in order for the artifact calibration of the Fluke 5700A to be valid and meet the manufacturer's absolute accuracy specifications. The uncertainty limit for the one-ohm resistor is ±10 ppm, and the uncertainty limit for the 10-kohm resistor is ±4 ppm.[2]

Later, we will compare the uncertainty budgets of these two examples to the uncertainty that can occur when using standard resistors.

What are Alpha and Beta Coefficients?

A key factor to accurate calibration is understanding and applying the alpha and beta coefficients for your standard resistors. Alpha and beta are mathematical constants associated with standard resistors that define a resistance change per degree of change in temperature. The importance of alpha and beta coefficients vary depending upon the magnitude of the coefficients for a given resistor, the amount of environmental temperature offset (difference between temperature where resistors is used and the temperature at which they were calibrated, the reference temperature), and the variance of the temperature. These factors are compared to the size of the uncertainty budget for a given calibration process.

The values of most physical properties vary with temperature, and resistivity is no exception. The relation between temperature and resistance for alloys that are used in standard resistors such as magnanin (copper, nickel, and manganese) and evanohm (nickel, chromium, copper, and aluminum) are well documented. The change in resistance due to temperature for standard resistors is generally given in either of the two following forms: [3]

$$R_t = R_{ref}\ 1+\alpha(t-t_{ref})+\beta(t-t_{ref})^2 \qquad (1)$$

$$R_t = R_{ref}\ 1+\alpha(t-t_{ref}) \qquad (2)$$

where

R_t = resistance at a measured temperature

t = temperature in degrees Celsius

R_{ref} = resistance of standard resistor at the reference temperature

t_{ref} = reference temperature

α = alpha coefficient in ppm*/deg C

β = beta coefficient ppm*/deg C^2

(* Some manufacturers use scientific notation instead of ppm in the definition of alpha and beta coefficients.)

Generally, equation (1) is more commonly used than equation (2). The correct equation to use with a given standard resistor is provided in documentation by the manufacturer of the resistor. When the standard resistor is used at any temperature other than R_{ref}, the value of R_t should always be computed at least during the development of the calibration process in order to determine whether the resistance change is significant to the overall calibration process. t_{ref} should also be provided by the manufacturer or calibration service provider and is usually either 23 or 25 degrees Celsius. R_{ref} is the measured value of the standard resistor at the reference temperature that is provided on the calibration report.

Typically, the alpha and beta coefficients need to be measured for each individual resistor. Although the alpha and beta coefficients will be similar for resistors of the same model and manufacturer, they generally are not exactly the same. The alpha and beta coefficients are dependent on the alloy used in the standard resistor, the construction of the resistor, and the variances of the alloy and construction throughout the manufacturing process. Typical values for resistors manufactured by companies such as Leeds and Northrup and Otto Wolf are 6 ppm/deg C for alpha and –0.5 ppm/deg C^2 for beta. Several companies are now producing standard resistors with much lower alpha and beta coefficients that are in the range of –0.03 ppm/deg C for alpha and -0.03 ppm/deg C^2 for beta.

In order to aid in determining whether temperature offsets or variance is significant to your process, the following examples have been developed:

Example 1, HP 3458A Calibration

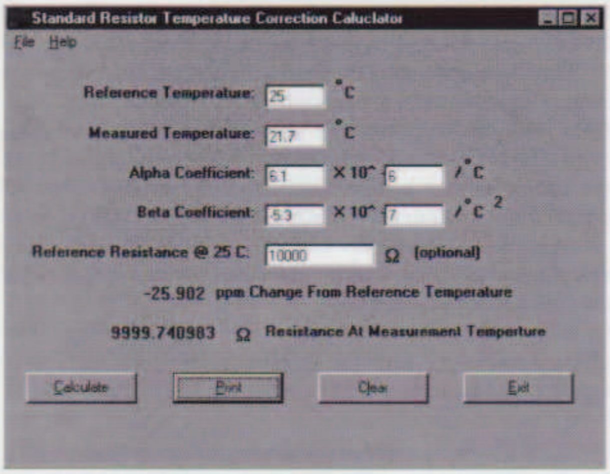
The laboratory is using an L&N 4040B 10,000 ohm resistor to calibrate a HP 3458A. The alpha coefficient is 6.1 ppm/deg C and the beta coefficient is –0.53 ppm/deg C^2. The reference temperature for the 4040B is 77 deg F (25 deg C) and the laboratory temperature at time of use is 71 deg F (21.7 deg C). The reference value of resistance for the L&N 4040B was used in the calibration of the HP 3458A. How much error was added to the calibration because of the temperature offset?

Using formula (1):

$$R_t = R_{ref}[1+6.1\frac{ppm}{\deg C}(21.7\deg C-25\deg C)-0.53\frac{ppm}{\deg C^2}(21.7\deg C-25\deg C)^2]$$

$$R_t = R_{ref}[1+6.1\frac{ppm}{\deg C}(-3.3\deg C)-0.53\frac{ppm}{\deg C^2}(-3.3\deg C)^2]$$

$$R_t = R_{ref}[1-(20.13ppm)+(5.77ppm)]$$



Reference Temperature: 25 °C
Measured Temperature: 21.7 °C
Alpha Coefficient: 6.1 X 10^ 6 /°C
Beta Coefficient: -5.3 X 10^ 7 /°C 2
Reference Resistance @ 25 C: 10000 Ω (optional)
-25.902 ppm Change From Reference Temperature
9999.740983 Ω Resistance At Measurement Temperture

Figure 1. Example 1 illustrated with Standard Resistor Temperature Correction Calculator, a freeware program developed by Verizon Electronic Repair Services.

The computations show that not correcting for the temperature offset has induced 25.9 ppm of error into the calibration process. If the metrologist used the reference calibration value of the resistor for this calibration process, they have unknowingly added an error into the process that is equal to 2.5 times the one year relative accuracy specification of the 10-kohm range! Even more importantly, this error does not account for the calibration uncertainty of the resistor or the random error for the process. When all uncertainties/errors are combined, the overall accuracy is almost four times the manufacturer's specification for the HP 3458A at the 10-kohm range. Has this fact been documented on the metrologist's certificate of calibration? Would the end user of the HP 3458A know this? Has an adequate calibration been performed on the HP 3458A?

Example 2, Fluke 5700A Artifact Calibration

The laboratory is using the same L&N 4040B to perform the artifact calibration of a Fluke 5700A. The mean laboratory temperature is 77 deg F, and varies ±3 deg F throughout the calibration process. Performing the computations for the variance of ±3 deg F in the same manner as Example 1, the worst case computed variability of the value of the standard resistor is approximately 11.9 ppm ($t - t_{ref}$ = 1.7 deg C). Once again, the metrologist who has not taken this into account has exceeded the uncertainty requirements for the standard resistor by nearly 300%, and this is before including the calibration uncertainty of the standard resistor in the uncertainty budget! Adding this much error into the calibration process voids the accuracy specifications for the Fluke 5700A. In order to compute the accuracy of the Fluke 5700A after such a calibration has taken place, the metrologist would have to use the supplemental information for the Fluke 5700A provided in the accuracy specifications and add the additional uncertainty as directed by the manufacturer.

Establishing the Alpha and Beta Coefficients

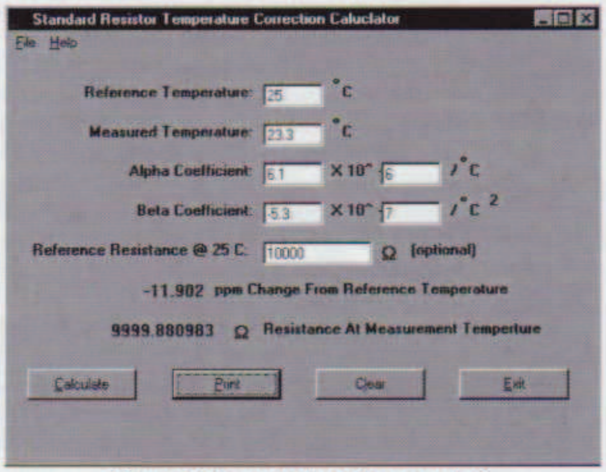
Okay, now I understand the importance of alpha and beta coefficients, but what if I don't know what they are for my standard resistors?

If you don't know what your alpha and beta coefficients are for your standard resistor, the first thing you should do is dig deep into your standard resistor history files. The manufacturer should have provided the alpha and beta coefficients with the original documents received when the standard was purchased. Once established, the alpha and beta coefficients are good for the life of the standard resistor.[3]

If you have lost the original documents or purchased the standard resistor from a used equipment dealer and have no information on the alpha and beta coefficients, you have two options. The first option is to check with your calibration service provider about having the alpha and beta coefficients established for your standard resistor through a special measurement process. Several laboratories that provide high accuracy measurements of standard resistors also have the ability to perform special tests such as the establishment of alpha and beta coefficients. The second option is to purchase new resistors with alpha and beta coefficients established.

Suggestions for Improved Performance in Example 1

There are several ways to improve the measurement process for Example 1. They will be covered from the least expensive improvement to the most expensive.



Reference Temperature: 25 °C
Measured Temprature: 23.3 °C
Alpha Coefficient: 6.1 X 10^ 6 /°C
Beta Coefficient: -5.3 X 10^ 7 /°C 2
Reference Resistance @ 25 C: 10000 Ω (optional)
-11.902 ppm Change From Reference Temperature
9999.880983 Ω Resistance At Measurement Temperture

Figure 2. Computations for example 2.

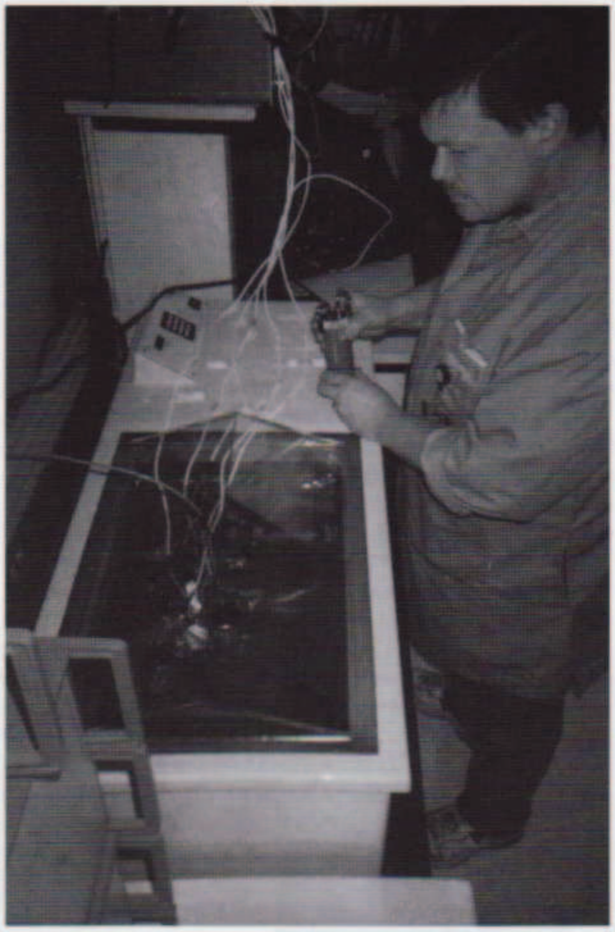


Figure 3. Dan Rumbold, Senior Metrology Technician at Verizon, utilizes a high quality temperature bath for the measurement of standard resistors

The least expensive method of improvement is to measure the temperature of the resistor at time of test and use the corrected value for the standard resistor. When using this method, one still has to consider the uncertainty associated with the temperature measurement, but it is usually much smaller than the error that is being corrected for. As an example, if the temperature of the resistor is measured using a thermometer with an accuracy of ± 0.25 deg C, the resistor could be corrected for the 25.9 ppm offset with an associated uncertainty of approximately 1.6 ppm, which should fall into an acceptable range for the uncertainty budget. (1.6 ppm is derived by using formula (1) and $t - t_{ref} = 0.25$ deg C.)

An alternative method is to purchase new standard resistors with much lower alpha and beta coefficients. If values characteristic of these resistors were used in Example 1, such as –0.02 ppm/deg C for alpha and –0.026 ppm/deg C^2 for beta, the resulting change in the standard resistor would only have been approximately 0.22 ppm, which again is usually in the acceptable range for one's uncertainty budget in this type of problem.

The most expensive method is to keep the old standard resistors and purchase a high quality temperature bath that maintains the desired temperature with minimum variability (Figure 3). The temperature bath could be set to the reference temperature and the resulting offset would become nearly zero. Because most high-level standards laboratories have a great deal of history associated with their older standard resistors, this is the method that they employ.

Suggestions for Improved Performance in Example 2

As in Example 1, there are several ways to improve the measurement process for Example 2. These will also be covered from least expensive to most expensive.

It is very inexpensive to construct a simple temperature lag bath. A temperature lag bath can consist of items as simple as a five-gallon bucket and white mineral oil. I would suggest that the mineral oil be purchased from a quality source, such as a company that sells temperature baths or standard resistors. It is important to cover the bucket so dust will not contaminate the oil making it resistive, which will cause measurement problems.

A bath constructed such as the one shown in the photo (Figure 4) will have a short-term stability well under 0.1 degree C. By entering 0.1 degree as the stability measurement into Example 2, the uncertainty associated with temperature variability decreases to approximately 0.62 ppm. It is important to note that a lag bath such as

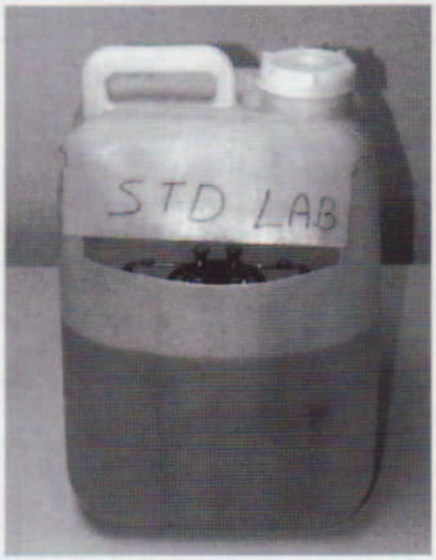


Figure 4. Simple temperature lag bath with a submerged standard resistor

this does not control the mean temperature, so the metrologist may still have to apply temperature offset corrections such as those found in Example 1 to determine the total process uncertainty.

For Example 2, one can also purchase new resistors with lower alpha and beta coefficients. If values characteristic of these resistors were used in Example 2, such as –0.02 ppm/deg C for alpha and –0.026 ppm/deg C^2 for beta, the resulting uncertainty due to the temperature variability would only have been approximately 0.04 ppm.

Lastly, one can also keep the old standard resistors and purchase a high-quality temperature bath that maintains the desired temperature with very little variability. A variability specification for a common temperature bath used with standard resistors is ±0.07 deg C. For Example 2, this would result in an uncertainty due to variability of only approximately 0.4 ppm.

Conclusion

Alpha and beta coefficients are a vital characteristic of standard resistors and their use is critical in some calibration processes. Not accounting for the change in resistance due to temperature change and variability can have significant effects on the measurement process, often exceeding the entire accuracy specification for the instrument that the standard resistor is intended to calibrate.

The methods shown for improving the uncertainty of the measurement process are not all-inclusive. These suggestions are merely simple ways to improve one's process, sometimes tenfold from the original measurement uncertainty. The basic methodology behind these suggestions can be further utilized to improve one's process to an extremely high level of accuracy. The techniques suggested could be improved upon and combined to determine temperature offset and variability to less than 0.01 degrees C, which results in reducing uncertainties to the order of 0.01 ppm.

In Part II of this series, we'll take a look at how the amount of current applied to standard resistors affects the measured value of the standard resistor. Understanding the effects varying currents in standard resistors will further enhance the accuracy of using standard resistors in the calibration process.

References

1. HP 3458A Specifications, Appendix A, HP 3458A Operating, Programming, and Configuration Manual, HP part number 03458-9004, February 1994 Edition 3
2. Fluke 5700A/5720A Series II Multi Function Calibrator, Operators Manual, Fluke Part number 601622, May 1996.
3. David Braudaway, "Precision Resistors: A Review of Material Characteristics, Resistor Design, and Construction Practices" IEEE Trans. on Inst. and Meas., Vol. 48, No. 5 (ISSN 0018-9456) pp 878-883.

The HP 3458A and Fluke 5700A are only used as examples. The use of these examples does not constitute an endorsement of these products by Verizon. Due to language inconsistencies throughout the referenced material, the words "Accuracy" and "Uncertainty" will be used interchangeably in this paper.

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