Assessment of Compliance with Specifications

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It is often the case that it will be necessary to provide information in calibration reports about whether or not the stated results for general purpose test and measurement equipment comply with a given specification. For measurement standards it is likely that the measured value and expanded uncertainty will be of more interest to the user, and specification compliance is less relevant. All measurement results are subject to uncertainty, which therefore has to be taken into account when assessing whether or not the "true value" really does lie within the specification limits. Uncertainty evaluation is normally performed as described in the GUM [1] and related documents [2, 3]. It is sometimes assumed that if the test accuracy ratio (TAR) or test uncertainty ratio (TUR) is better than a certain value, say 4:1 or 10:1 — then the uncertainty can be ignored and, providing the stated result is within the specification limits, compliance can be assumed. This is, at best, over-simplistic and is usually demonstrably incorrect, as will be shown in this article.

Conventional Approach

A simple representation of the situation when a result (●), expanded by its uncertainty, is considered in relation to specification limits is shown in Figure 1.

Case 1
The result, extended by the uncertainty of measurement, lies within the specification limits. A statement of compliance can therefore be made for the confidence level stated.

Case 2
The result lies within the specification limits. However the uncertainty overlaps one specification limit and therefore a statement of compliance cannot be made for the confidence level stated. The result does, however, mean that compliance with the specification is more likely than non-compliance.

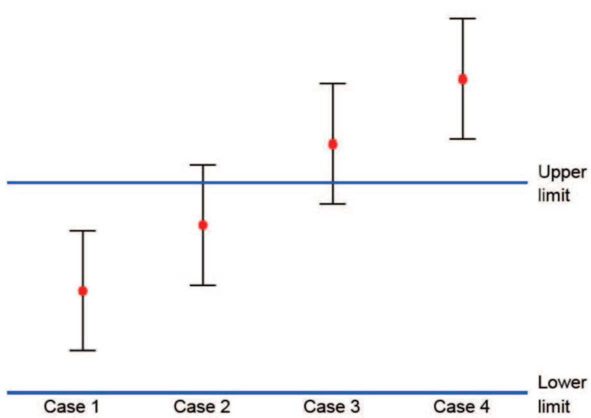

Figure 1. Representation of compliance and noncompliance when results are considered with uncertainties.

Case 3
The result lies outside the specification limits. However the uncertainty overlaps one specification limit and therefore a statement of non-compliance cannot be made for the confidence level stated. The result does, however, mean that non-compliance with the specification is more likely than compliance.

Case 4
The result, extended by the uncertainty of measurement, lies outside the specification limits. A statement of non-compliance can therefore be made for the confidence level stated.

This straightforward approach is certainly usable and is similar to that described in various publications on the subject, such as ILAC G8:1996 [4]. It is a simplification of the situation, although a practical and "safe" one.

Probability Distributions

The simplification arises because the uncertainty is depicted using simple "bars" but is, in fact, a probability distribution, normally Gaussian, as shown in Figure 2.

This means that consideration has to be given to the area of the distribution that is contained within the limits when performing a detailed assessment of compliance with the specification. Furthermore, the implication is that compliance - or non-compliance - can only be stated in conjunction with an associated confidence level. This is because there will always be a possibility of one, or both, of the tails of the distribution overlapping the limits.

As an expanded uncertainty is normally expressed for a coverage probability of 95.45% (coverage factor k = 2), it is generally accepted practice that statements regarding compliance will relate to the same level of confidence.

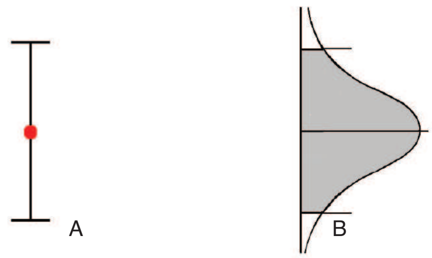

Figure 2. Probability distribution of uncertainty is normally Gaussian as in B, rather than a simple bar, as in A.

If this is the case for a given situation, then comparison of the expanded uncertainty using a coverage factor $k = 2$ with the specification limit is unduly pessimistic. It will yield a confidence level of 97.7% or greater. This is because only one tail of the distribution will usually be the subject of comparison with the limit. If, as should be the case, the uncertainty is small compared with the specification, the probability contained within the other tail will already be within the specification limits.

A new coverage factor, k_s, can therefore be used for the purpose of comparison with a specification limit. Assuming a normal distribution, the value of k_s required to achieve at least 95.45% confidence is 1.69.

This also means that it is possible to evaluate compliance, or non-compliance, at a different level of confidence. Such an approach should, of course, be taken with the agreement and understanding of the customer. This procedure may also be used in cases where a customer has requested a compliance statement for other confidence levels.

Two items of information are needed to deduce the confidence level at which compliance can be stated:

- The combined standard uncertainty $u_c(y)$
- The difference between the specification limit and the result, $L_S - y$

The probability of compliance when the result lies within the specification, or that of non-compliance when it lies outside the specification, can be obtained from Table 1.

Normally $\left|\frac{L_S - y}{u_c(y)}\right|$ will not be an integer and it will be necessary to interpolate between the values given in the table. Linear interpolation will suffice for $\left|\frac{L_S - y}{u_c(y)}\right|$ <2; higher-order interpolation should be used otherwise. Alternatively, the next lower value may be used.

It should be noted that this procedure is only valid when the uncertainty breaches *one* of the specification limits and for this reason the uncertainty should be sufficiently small that an insignificant portion of the distribution approaches the other limit. Furthermore, as the result approaches either limit there will come a point at which no reasonable decision can be made regarding compliance or non-compliance with the specification. In the extreme, if the result coincided exactly with one of the limits, there would always be 50% confidence in the decision, regardless of the magnitude of the uncertainty. For this reason, and by general convention, the data in Table 1 are limited to a confidence probability of 70% and above.

Example

A measurement yields a result y of 0.80 units with a combined standard uncertainty $u_c(y)$ of 0.15 units. The specification is ± 1.00 unit. At what confidence level can

Probability of compliance or non-compliance (%)	$\left\|\frac{L_S - y}{u_c(y)}\right\|$	Probability of compliance or non-compliance (%)	$\left\|\frac{L_S - y}{u_c(y)}\right\|$	Probability of compliance or non-compliance (%)	$\left\|\frac{L_S - y}{u_c(y)}\right\|$
99.9	3.29	91	1.34	80	0.84
99.73	2.78	90	1.28	79	0.81
99	2.32	89	1.23	78	0.77
98	2.05	88	1.17	77	0.74
97	1.88	87	1.13	76	0.71
96	1.75	86	1.08	75	0.68
95.45	1.69	85	1.04	74	0.64
95	1.64	84	1.00	73	0.61
94	1.56	83	0.95	72	0.58
93	1.48	82	0.92	71	0.55
92	1.41	81	0.88	70	0.52

NOTE: This data were produced using a Monte Carlo Simulation process whereby a Gaussian distribution was generated and the associated amount of probability within a defined limit, corresponding to the specification, was determined. 10^6 MCS trials were used for each value and the results were rounded to two decimal places.

Table 1. Probability of compliance.

compliance with the specification be made?

$$\left|\frac{L_S - y}{u_c(y)}\right| = \left|\frac{1.00 - 0.80}{0.15}\right| = 1.33$$

The next lower value in the table is 1.28, therefore it has been demonstrated that the specification is met for at least 90% confidence.

Specification Limits – Further Considerations

Specification limits are usually treated as absolute, analogous to a rectangular probability distribution. This may not always be the case, there are situations where the specification is characterised by a normal distribution. It is stated in the GUM [1] that, when a specification is quoted for a given coverage probability, then a normal distribution can be assumed. Some, but not many, manufacturers state confidence levels for their equipment specifications.

If both the uncertainty U and the specification L are stated at the same coverage probability (confidence level), then the specification is demonstrated to be met under those conditions when

$y < \sqrt{L_S^2 - U^2}$ and is demonstrated to be failed when $y > \sqrt{L_S^2 + U^2}$, where

y = reported result
U = expanded uncertainty
L_S = specification limit

Example

A digital multimeter is calibrated with an applied voltage of 10 .000 000 V dc. The expanded uncertainty, U, is ±3 ppm (95.45% coverage probability, k=2) and the reading is + 7.0 ppm from the nominal value. The manufacturer's specification for this reading is stated as 10 ppm at 99% confidence. Is the specification met?

The comparison with specification has to be carried out with both the specification and the uncertainty at the same coverage probability. The specification at 95.45% coverage probability, L95.45, can be obtained from

$$L_{95.45} = L_{99} \cdot \frac{k_{95.45}}{k_{99}}$$

where $k_{95.45}$ and k_{99} are the coverage factors for 95.45% and 99% confidence respectively, as obtained from the *t*-distribution. This gives values of $k_{95.45}$ = 2.0 and k_{99} = 2.58, therefore

$$L_{95.45} = 10 \cdot \frac{2.0}{2.58} = 7.75 \text{ ppm.}$$

Now, $\sqrt{L_{95.45}^2 - U_{95.45}^2} = \sqrt{7.75^2 - 3^2} =$ 7.15 ppm

As 7.0 < 7.15, then compliance with the specification has been demonstrated, taking the measurement uncertainty into account.

Test-Uncertainty Ratios

The discussions so far reveal that there will be a range inside the specification limits within which the reported values have to lie in order to ensure that compliance at a stated confidence level can be assured. This is referred to as the "acceptance zone" in Figure 3 below. The width of the acceptance zone with respect to the specification limits depends on the expanded uncertainty and the level of confidence required.

For a normal distribution and a coverage probability of 95.45%, the combined standard uncertainty $u_c(y)$ is one-half of the expanded uncertainty U. If a test-uncertainty ratio (TUR) were to be used as the basis for compliance assessment, the extent of the acceptance zone can be calculated from this and from the information in Table 1.

For example, suppose a TUR of 4:1 is used. This means that the ratio of the specification to $u_c(y)$ will be 4: $\frac{U}{2}$, i.e. 8:1. So, the combined standard uncertainty is one eighth of the specification, i.e. $0.125L_S$.

From Table 1, $\left|\frac{L_S - y}{u_c(y)}\right|$ = 1.69 for 95.45% confidence in the compliance decision. Rearranging and substituting gives

$L_S - y$ = 1.69 x 0.125 = 0.21.

In other words, the reported value y has to be 21% or more away from the specification limit to demonstrate compliance with that limit for at least 95.45% confidence. The same, of course, applies to the other specification limit. So the result must lie within the central 58% of the specification limits to ensure that compliance has been demonstrated for 95.45% confidence or better. This represents the acceptance zone in Figure 3.

It follows that if a TUR of 4:1 is used, and no consideration is given to the acceptance zone, there will only be (58 x 0.9545)% confidence that compliance with the specification has been obtained, i.e. 55%. This and other values of confidence in compliance statements are presented in Table 2.

This clearly demonstrates that reliance on test-uncertainty ratios

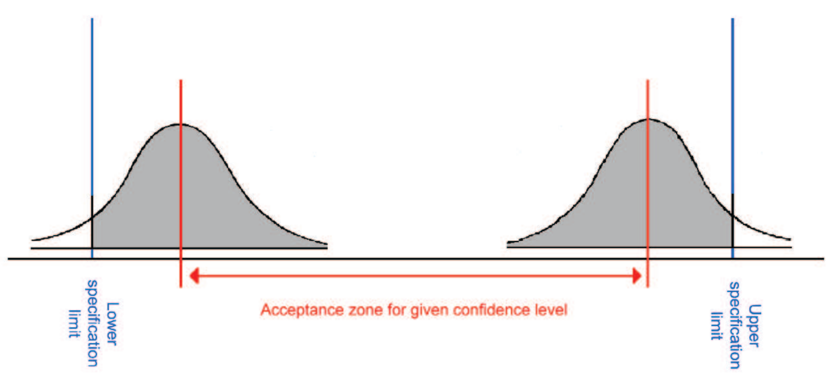

Figure 3. The acceptance zone is a range inside the specification limits.

TUR (Uncertainty U is for a coverage probability of 95.45%)	Confidence in compliance with specification (%)		TUR (Uncertainty U is for a coverage probability of 95.45%)	Confidence in compliance with specification (%)
4:1	55		10:1	79
5:1	63		20:1	88
6:1	72		50:1	92
7:1	69		75:1	94
8:1	75		100:1	95.41
9:1	77		∞	95.45

Table 2. Confidence values for compliance with coverage probability of 95.45%.

TUR	Confidence in compliance with specification (%)		TUR	Confidence in compliance with specification (%)
4:1	96.8		9:1	99.4
5:1	97.9		10:1	99.5
6:1	98.6		15:1	99.8
7:1	99.0		20:1	99.9
8:1	99.2		50:1	100.0

Table 3. Confidence values for Gaussian distribution when coverage probability is 95.45%.

alone cannot possibly give a reasonable level of confidence in compliance statements, unless the TUR is extremely high (>100:1).

The situation is improved, however, if the specification itself is described as a Gaussian distribution. As noted earlier, such a specification is demonstrated to be met when $y < \sqrt{L_S^2 - U^2}$. If, for example, a 4:1 TUR is substituted for L_S and U, y has to lie within 0.968 of the specification limits to demonstrate that the specification is met. In other words, there will be 96.8% confidence that compliance with the specification has been achieved if the TUR alone is used as the basis for comparison.

Table 3 shows the confidence that this and other TURs yield when used as the basis for compliance assessment when both the specification and the uncertainty are described by Gaussian distributions at the same confidence level.

These levels of confidence in the compliance assessment using TURs are much more acceptable than those obtained when the specification is described as simple limit values. It is unfortunate that "limit values" are usually the case and that so few manufacturers give confidence levels associated with their specifications.

Conclusions

A proper evaluation of specification compliance requires consideration of the nature of the probability distribution associated with the assigned uncertainty. A simple comparison of the result and its uncertainty with the limits may suffice but reliance on test-uncertainty ratios alone can easily result in poor decisions being made. A correct analysis of compliance must consider the amount of probability due to the uncertainty that is contained within the specification limits. In this context, it could be argued that there is no place for TURs in compliance evaluation, particularly as very few manufacturers provide information about the confidence levels associated with their specifications.

References

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